1.

(a) Charge is distributed in space according to $\rho = \rho_0 \cos \alpha x \cos \beta y \cos \gamma z$ forming an infinite spatial lattice. Find the electrostatic potential $\Phi(x, y, z)$.

(b) Charge is distributed over a sphere of radius R with surface charge density $\sigma = \sigma_0 \cos^2 \theta$. Find the electrostatic potential $\Phi(r, \theta, \phi)$ using multipole expansion.

2. Charge Q is uniformly distributed throughout the volume of a sphere of radius R.

(a) Find the electric field $\boldsymbol{E}$ for all points $\boldsymbol{r}$ inside and outside the sphere.

(b) Find the electrostatic potential Φ for all points $\boldsymbol{r}$ inside and outside the sphere.

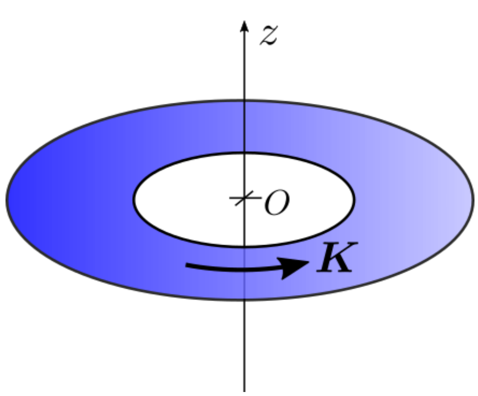
(c) Find the amount of work W needed to assemble this charge distribution.

3. Parallel plate capacitor is filled with the dielectric whose permittivity varies along the axis x perpendicular to the plates according to $\varepsilon = \varepsilon_0(x + d)/d$, where the plates are located at $x = 0$, $x = d$ and each have area A. Assume the plates are large enough that boundary effects are negligible.

(a) Find the capacitance C of the capacitor.

(b) Find the distribution of the bound charges when a potential difference V is applied to the plates: $\Phi(0) - \Phi(d) = V$.

4.



A thin disc of outer radius b with a concentric hole of radius a carries a current given by surface current density $\boldsymbol{K} = K_0 \hat{\boldsymbol{\phi}}$ around its axis $\hat{\boldsymbol{z}}$.

(a) Find the total current I.

(b) Find the magnetic field $\boldsymbol{B}$ (magnitude and direction) at the center O of the disk.

(c) Find the magnetic dipole moment $\boldsymbol{m}$ of the disk.

5.

A fast (but nonrelativistic) electron is passing through the gap between two oppositely charged parallel plates in the direction parallel to the plates. The potential between the plates is V, the distance between the plates is d and their length along the direction of the electron velocity is ℓ.

The charge of the electron is q_e and the mass is m_e.

(a) Find the condition on the velocity of the electron (v much larger or smaller than what?) so that the trajectory of the electron can be still considered approximately as a straight line.

(b) Find the total amount of electromagnetic radiation emitted by the electron under this condition.

(c) Draw the directional dependence of this radiation. What is the direction (or directions) in which the radiation is minimal?

Equations

$$\boldsymbol{\nabla}\cdot\boldsymbol{D}=\rho;\quad \boldsymbol{\nabla}\times\boldsymbol{E}=-d\boldsymbol{B}/dt;\quad \boldsymbol{\nabla}\times\boldsymbol{H}=\boldsymbol{J}+d\boldsymbol{D}/dt;\quad \boldsymbol{\nabla}\cdot\boldsymbol{B}=0;$$

$$\boldsymbol{D}=\varepsilon_0\boldsymbol{E}+\boldsymbol{P};\quad \boldsymbol{H}=\boldsymbol{B}/\mu_0-\boldsymbol{M};\quad \boldsymbol{E}=-\boldsymbol{\nabla}\Phi-\partial\boldsymbol{A}/\partial t;\quad \boldsymbol{B}=\boldsymbol{\nabla}\times\boldsymbol{A};$$

$$\Phi=\frac{1}{4\pi\varepsilon_0}\int d^3\boldsymbol{r}'\frac{\rho(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|};\quad \boldsymbol{A}=\frac{\mu_0}{4\pi}\int d^3\boldsymbol{r}'\frac{\boldsymbol{J}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|};$$

$$U=\frac{1}{2}\int d^3\boldsymbol{r}\,\rho\,\Phi;\quad \boldsymbol{p}=\int d^3\boldsymbol{r}\,\rho\boldsymbol{r};\quad \boldsymbol{m}=\frac{1}{2}\int d^3\boldsymbol{r}\,\boldsymbol{r}\times\boldsymbol{J};$$

$$\frac{1}{|\boldsymbol{x}-\boldsymbol{y}|}=\sum_l\frac{r_<^l}{r_>^{l+1}}P_l(\cos\gamma)=\sum_{lm}\frac{4\pi}{2l+1}\frac{r_<^l}{r_>^{l+1}}Y_{lm}(\Omega_{\boldsymbol{x}})Y_{lm}^*(\Omega_{\boldsymbol{y}})$$

$$Y_{00}\sim 1;\qquad Y_{10}\sim\cos\theta;\qquad Y_{20}\sim 3\cos^2\theta-1;$$
$$Y_{11}\sim\sin\theta\,e^{i\phi};\qquad Y_{21}\sim\cos\theta\sin\theta\,e^{i\phi};$$
$$Y_{22}\sim\sin^2\theta\,e^{2i\phi}.$$

$$\mathcal{P}=\frac{cq^2a^2}{6\pi\varepsilon_0}$$