University of Illinois at Chicago
Department of Physics

Thermal Physics and Statistical Mechanics Qualifying Examination

January 6, 2014
9.00 am – 12:00 pm

Full credit can be achieved from completely correct answers to 4 questions. If the student attempts all 5 questions, all of the answers will be graded, and the top 4 scores will be counted toward the exam's total score.

Problem 1.

a) Using the definitions of c_P and c_V and the first law of thermodynamics derive the general relation:

$$c_P - c_V = \left[P + \left(\frac{\partial E}{\partial V}\right)_T\right]\left(\frac{\partial V}{\partial T}\right)_P,$$

where c_P and c_V are the specific heat capacities at constant pressure and volume, respectively, and E and V are the internal energy and volume of 1 mol.

b) From the fundamental differential relation for dE and the differential relation for Helmholtz free energy $dF = -SdT - PdV$ show that for a constant number of particles:

$$\left(\frac{\partial E}{\partial V}\right)_T + P = T\left(\frac{\partial P}{\partial T}\right)_V$$

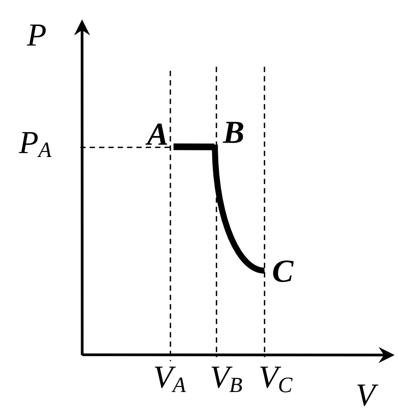
c) Use the above results to find $c_P - c_V$ for Ideal Gas

d) Use the above results to find $c_P - c_V$ a Van der Waals gas $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

e) Show that at a constant pressure in the limit $V \to \infty$ the result for $c_P - c_V$ of a Van der Waals gas is the same as for $c_P - c_V$ of an ideal gas.

Problem 2.

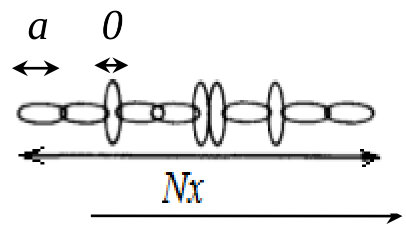
The initial state of monoatomic ideal gas is described by T_A, P_A and V_A (the temperature, pressure, and volume, respectively). The gas is taken over the path A→B→C quasistatically as shown in the sketch. The volumes are related as $V_B = 1.5V_A, V_C = 2V_A$. Find:



a) How much work does the gas do on the path A→B and what is the change in its internal energy?
b) How much heat is absorbed in going from A→B?
c) Derive the expression for the entropy change for an arbitrary process B→C.
d) If B→C is an adiabatic process, find the final gas pressure and the entropy change (from the general expression obtained in part *c*).

Problem 3.

Consider a one-dimensional chain consisting on N>>1 segments as illustrated in the sketch. Let the length of each segment be a when the long dimension is parallel to the chain and zero when the segment is vertical (i.e. long dimension is perpendicular to the chain direction). Each segment has just two states, horizontal and vertical, and each of these states is not degenerate. The distance between the chain ends is fixed.



a) For a given length $L=Nl$ $(0<l<a)$ of the chain what is the total number of microstates accessible by the system and what is the entropy of the system as a function of l?

b) Write down the appropriate thermodynamic identity for the system (equivalent to the first law) and describe (qualitatively) how one could obtain an expression for tension force F, necessary to maintain the length Nl (assuming the joints turn freely), from the result obtained in part a)
c) Obtain the relationship between that tension force F maintaining the distance Nl and the temperature T using the canonical ensemble description.
d) Under which conditions does your answer lead to Hooke's law and what is corresponding expression for the spring constant?

Problem 4.

A system of two energy levels E_0 and E_1 is populated by N particles at temperature T. Assume that $E_1>E_0$, so that $\Delta E=E_1-E_0$ is positive.

a) Derive an expression for the average energy per particle as function of temperature.
b) Determine the limiting behavior and value for average energy per particle in the limits of $T\to 0$ and $T\to\infty$.
c) Derive an expression for specific heat of the system.
d) Compute specific heat in the limits of $T\to 0$ and $T\to\infty$.

Problem 5.

Consider an ideal gas of N spin-½ fermions is confined to an area A in 2 dimensions. Consider the ground state ($T=0$) for such system.

a) Derive the expression for the number of single particle microstates in the momentum interval p, $p+dp$ for the given system.

b) Find the Fermi momentum and chemical potential of the system.

c) Find the average energy per particle for the system.

d) Now assume the gas system is placed in the uniform magnetic field H which makes an additional single particle energy contribution $\pm\mu_B H$ (depending on the spin orientation). Find the Fermi momenta for spin-up and spin-down fermions.

e) Calculate the average magnetization m per area for the system.