University of Illinois at Chicago
Department of Physics

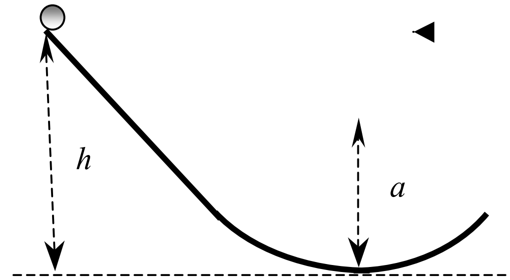
Classical Mechanics
Qualifying Examination

January 4, 2012
9.00 am – 12:00 pm

Full credit can be achieved from completely correct answers to 4 questions. If the student attempts all 5 questions, all of the answers will be graded, and the top 4 scores will be counted toward the exam's total score.

Problem 1.

A cylinder of a non-uniform radial density with mass M, length l and radius R rolls without slipping from rest down a ramp and onto a circular loop of radius a. The cylinder is initially at a height h above the bottom of the loop. At the bottom of the loop, the normal force on the cylinder is twice its weight.



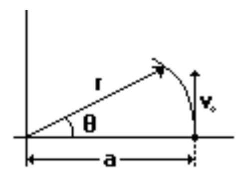
a) Expressing the rotational inertia of the non-uniform cylinder in the general form ($I=\beta MR^2$), express the β in terms of h and a.

b) Find numerical value of β if the radial density profile for the cylinder is given by $\rho(r) = \rho_2 r^2$;

c) If for the cylinder of the same total mass M the radial density profile is given by $\rho_n(r) = \rho_n r^n$, where $n \in 0,1,2,3...$, describe qualitatively how do you expect the value of β to change with increasing n. Explain your reasoning.

Problem 2.

A particle of unit mass is projected with a velocity v_0 at right angles to the radius vector at a distance a from the origin of a center of attractive force, given by $f(r) = -k\left(\frac{4}{r^3} + \frac{a^2}{r^5}\right)$

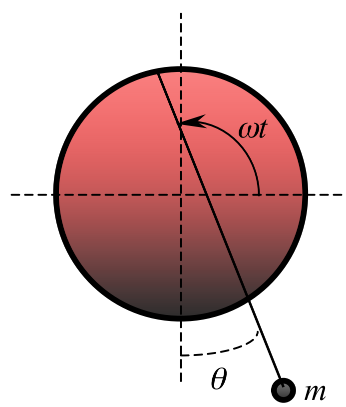


For initial velocity value given by $v_0^2 = \frac{9k}{2a^2}$, find the polar equation of the resulting orbit.

Problem 3.

A simple pendulum of length b and mass m is suspended from a point on the circumference of a thin massless disc of radius a that rotates with a constant angular velocity ω about its central axis. Using Lagrangian formalism, find

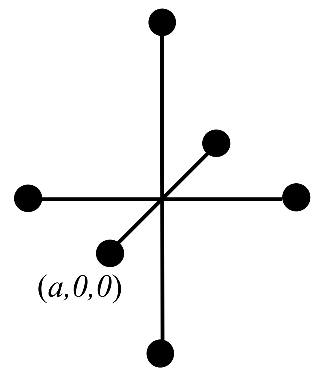
a) the equation of motion of the mass m;
b) the solution for the equation of motion for small oscillations.



Problem 4.

A rigid body consists of six particles, each of mass m, fixed to the ends of three light rods of length $2a$, $2b$, and $2c$ respectively, the rods being held mutually perpendicular to one another at their midpoints.

a) Write down the inertia tensor for the system in the coordinate axes defined by the rods;
b) Find angular momentum and the kinetic energy of the system when it is rotating with an angular velocity ω about an axis passing through the origin and the point (a,b,c).



Problem 5.

The force of a charged particle in an inertial reference frame in electric field $\vec{E}$ and magnetic field $\vec{B}$ is given by $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$, where q is the particle charge and $\vec{v}$ is the velocity of the particle in the inertial system.

a) Prove that the transformation from a fixed frame to a rotating frame is given by
$$\ddot{\vec{r}} = \ddot{\vec{r}}' + \dot{\vec{\omega}} \times \vec{r}' + 2\vec{\omega} \times \vec{v}' + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}'\right)$$
b) Find the differential equation of motion referred to a non-inertial coordinate system rotating with angular velocity $\vec{\omega} = -\left(q/2m\right)\vec{B}$, for small $\vec{B}$ (neglect B^2 and higher order terms).