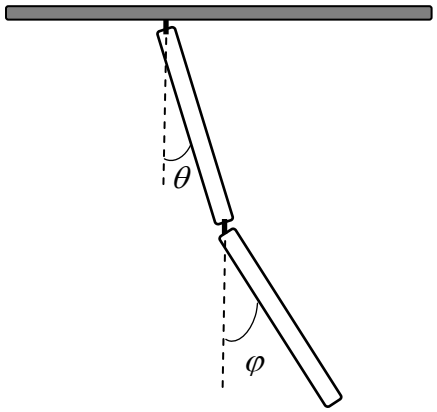
University of Illinois at Chicago
Department of Physics

Classical Mechanics
Qualifying Examination

January 4, 2011
9.00 am – 12:00 pm

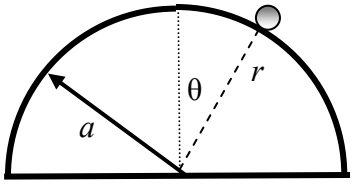
Full credit can be achieved from completely correct answers to 4 questions. If the student attempts all 5 questions, all of the answers will be graded, and the top 4 scores will be counted toward the exam's total score.

Problem **1.**



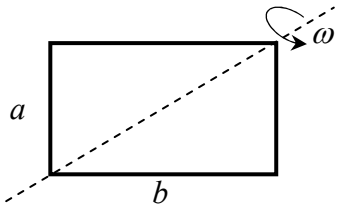
Two identical rods of mass m and length l are connected to the ceiling and together vertically by small flexible pieces of string. The system then forms a physical double pendulum. Find the frequencies of the normal modes of this system for small oscillations around the equilibrium position. Describe the motion of each of the normal modes.

Problem **2.**



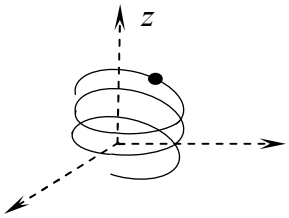
The particle is sliding down from the top of the hemisphere of radius a. Find: a) normal force exerted by the hemisphere on the particle; b) angle with respect to the vertical at which the particle will leave the hemisphere.

Problem **3.**



A uniform rectangular plane lamina of mass m and dimensions a and b (assume $b > a$) rotates with the constant angular velocity ω about a diagonal. Ignoring gravity, find: a) principal axes and moments of inertia; b) angular momentum vector in the body coordinate system; c) external torque necessary to sustain such rotation.

Problem **4.**



A particle of mass m moves frictionless under the influence of gravity along the helix $z = k\theta$, $r = const$, where k is a constant, and z is vertical. Find: a) the Lagrangian; b) the Hamiltonian. Determine: c) equations of motion.

Problem **5.**

A particle of mass m is bound by the linear potential $U = kr$, where $k = const$. Find:

a) For what energy and angular momentum will the orbit be a circle of radius r about the origin?
b) What is the frequency of this circular motion?
c) If the particle is slightly disturbed from this circular motion, what will be the frequency of small oscillations?