University of Illinois at Chicago

Department of Physics

Electricity and Magnetism

Qualifying Examination

Thursday, January 7, 2010

Full credit can be achieved from completely correct answers to 4 questions. If the student attempts all 5 questions, all of the answers will be graded, and the top 4 scores will be counted toward the exams total score.

Various equations, standard integrals, etc. are provided on the last page of the exam.

1. Charged sphere

The potential on the surface of a sphere of radius a as a function of polar angle θ is given by

$$\Phi(r,\theta)\big|_{r=a} = V_0 \cos 2\theta,$$

where V_0 is a known constant. Also given is that $\Phi \to 0$ as $r \to \infty$.

(a) Find the potential outside the sphere as a function of r ($r > a$) and θ.

(b) Find the total charge Q on or inside the sphere.

2. Hydrogen atom

The electric charge of electron is distributed in a hydrogen atom according to

$$\rho(\boldsymbol{r}) = -\frac{e_0}{\pi a^3}\, e^{-2r/a},$$

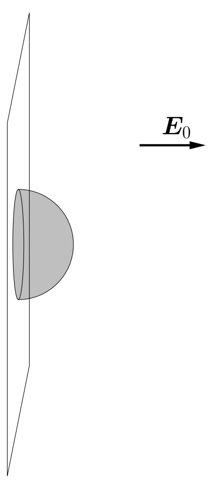
where e_0 and a are fundamental constants.

(a) Find the energy U of the electrostatic interaction of the electron with the nucleus (proton).

(b) Find the value of the electrostatic potential Φ created by the electron charge distribution $\rho(\boldsymbol{r})$ at the center of the atom $\boldsymbol{r} = 0$.

(c) Find the *total* electric field $\boldsymbol{E}$ created by the whole atom as a function of $\boldsymbol{r}$ for any $\boldsymbol{r}$. Determine the magnitude and the direction (inward or outward).

3. Dielectric bead

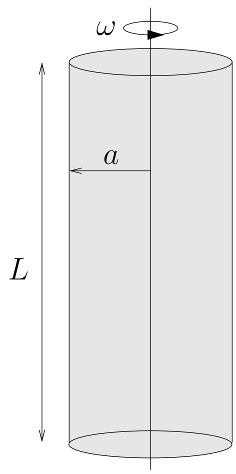


A hemispherical bead of radius a made of a dielectric material with dielectric permittivity ε is placed on an infinite conducting sheet. The electric field far away from the sheet is perpendicular to it and equals $\boldsymbol{E}_0$.

(a) Find electrostatic potential Φ everywhere to the right of the sheet (both inside and outside of the bead). *Hint*: look for the solution in the form $\Phi = R(r)\cos\theta$.

(b) Find the surface charge density σ at any given point on the sheet outside the bead $r > a$.

4. Magnetic field of a rotating cylinder



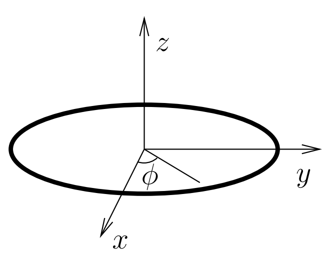
Electric charge is distributed uniformly with constant volume density α inside a very long cylinder, whose length L is much greater than its radius a: $L \gg a$. The cylinder is rotated around its axis with angular velocity ω.

(a) What is the current density $\boldsymbol{J}$ at any given point inside the cylinder? Express the result in cylindrical coordinates, i.e., determine J_ρ, J_ϕ, J_z as functions of ρ, ϕ and z.

(b) What is the magnetic field at any given point *outside* the cylinder, i.e., at distances ρ, such that $\rho > a$ (but $\rho \ll L$). Express the result in cylindrical coordinates, i.e., determine B_ρ, B_ϕ, B_z as functions of ρ, ϕ and z.

(c) What is the magnetic field at any given point *inside* the cylinder. Express the result in cylindrical coordinates, i.e., determine B_ρ, B_ϕ, B_z as functions of ρ, ϕ and z.

5. Radiating ring



A ring of wire of radius a carries current I which varies with time t and angle ϕ along the ring according to $I = I_0 \sin\phi \cos\omega t$.

(a) Find the rate of change $d\boldsymbol{p}/dt$ of the electric dipole moment of the ring at time t.

(b) Find the rate of change $d\boldsymbol{m}/dt$ of the magnetic dipole moment of the ring at time t.

(c) In dipole approximation, what is the polarization (e.g., circular, linear, none) of radiation emitted along *each* of the directions x, y and z? For each linearly polarized case indicate the orientation of the polarization axis (e.g., x or y or z). Organize your answer in a table like this:

radiation direction	polarization type	polarization axis (if linear)
x	...	...
y	...	...
z	...	...

Equations

$$\boldsymbol{\nabla}\cdot\boldsymbol{D}=\rho;\quad \boldsymbol{\nabla}\times\boldsymbol{E}=-d\boldsymbol{B}/dt;\quad \boldsymbol{\nabla}\times\boldsymbol{H}=\boldsymbol{J}+d\boldsymbol{D}/dt;\quad \boldsymbol{\nabla}\cdot\boldsymbol{B}=0;$$

$$\partial\rho/\partial t+\boldsymbol{\nabla}\cdot\boldsymbol{J}=0;$$

$$\boldsymbol{D}=\varepsilon\boldsymbol{E};\quad \boldsymbol{B}=\mu\boldsymbol{H};$$

$$\boldsymbol{E}=-\boldsymbol{\nabla}\Phi-\partial\boldsymbol{A}/\partial t;\quad \boldsymbol{B}=\boldsymbol{\nabla}\times\boldsymbol{A};$$

$$U=\int d^3\boldsymbol{r}\,\rho\,\Phi$$

$$\boldsymbol{p}=\int d^3\boldsymbol{r}\,\rho\boldsymbol{r};\quad \boldsymbol{m}=\frac{1}{2}\int d^3\boldsymbol{r}\,\boldsymbol{r}\times\boldsymbol{J};$$

$$\Phi=\sum_l(A_l r^l+B_l/r^{l+1})P_l(\cos\theta);$$

$$P_0=1;\quad P_1=x;\quad P_2=\frac{1}{2}(3x^2-1);$$

$$\int xe^{-x}dx=-e^{-x}(1+x);\quad \int x^2e^{-x}dx=-e^{-x}(2+2x+x^2).$$