University of Illinois at Chicago

Department of Physics

Classical Mechanics

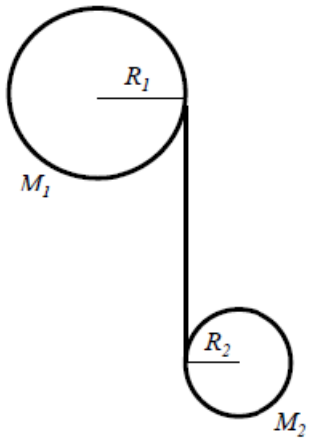
Qualifying Examination

Tuesday, January 5, 2010
9:00 am – 12:00 pm

Full credit can be achieved from completely correct answers to 4 questions. If the student attempts all 5 questions, all of the answers will be graded, and the top 4 scores will be counted toward the exam's total score.

Problem 1

A frictionless pulley, constructed from a solid disk of mass M_1 and radius R_1, can rotate about its horizontal axis of rotation. A string is wound around the pulley, with its other end wound around a second pulley of mass M_2 and radius R_2 that is falling downwards while maintaining the horizontal orientation of its axis. Assuming that the string is massless, does not slip, and remains vertical and taut during the motion, find:



(a) the linear acceleration of the center of mass of the pulley M_2

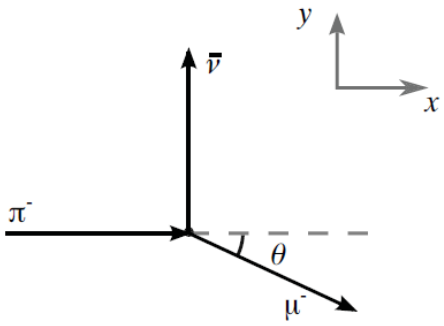
(b) the angular acceleration of the pulley M_2

(c) the angular acceleration of the pulley M_1

(d) the tension in the string

Problem 2

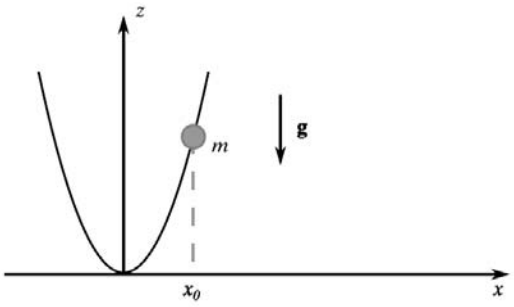
a) A pion (π^-) is moving with a velocity *v* and decays into a muon (μ^-) and an antineutrino ($\bar{\nu}_\mu$). If the antineutrino moves after the decay perpendicular to the direction of the π^-, find the energy of the muon and the angle θ of the muon's direction relative to the π^- in terms of: the π^- mass m_π, the muon mass m_μ, $\beta_\pi = \frac{v}{c}$, and $\gamma_\pi = 1/\sqrt{1-\beta_\pi^2}$. (For simplicity you can assume that the speed of light c=1.)



b) If the π^- is at rest when it decays into a muon and an antineutrino, find the distance traveled by the muon before it decays (i.e., during its lifetime τ) in terms of: m_π, m_μ, and the muon lifetime τ.

Problem 3

A bead of mass m slides under gravity along a smooth vertical parabolic wire. The shape of the wire is given by the equation $ax^2-z=0$. The bead starts from rest at $x=x_0$.



(a) Write the Lagrangian of the system.

(b) Use the Lagrange multiplier method to determine the force that the wire exerts on the bead as a function of x.

Problem 4

A point particle of mass m is moving under the potential

$$V(x,y) = \frac{k}{2}\sin^2\sqrt{x^2+y^2-xy}$$

where *k* is a positive constant.

(a) Write the Lagrangian of the system.

(b) Prove that the origin *x=y=0* is a stable equilibrium point and write the Lagrangian appropriate for small oscillations about this point.

(c) Find the normal frequencies of the system.

(d) Construct the normal coordinates of the system and express the Lagrangian in terms of these coordinates.

Problem 5

An object of mass m is thrown vertically upward from the earth's surface with initial speed v_0. There are only two forces acting on the object: its weight and the air resistance which is opposite to the direction of motion and has a magnitude of kmv^2, where k is a positive constant and v is the object's speed at time t.

a) Find the maximum height H reached by the mass as a function of: k, v_0, and the acceleration of gravity g.

b) After the mass has reached the maximum height H, it starts falling down. If $kv_0^2 < g$, find the distance the mass has dropped from its maximum height when it reaches speed v_0 as a function of: k and H.