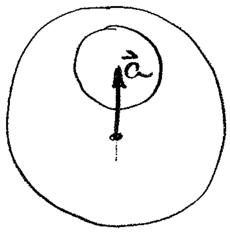
1.



(a) Gauss: $4\pi r^2 E_{out} = \frac{4\pi}{3} R^3 \rho / \varepsilon_0$

$$\vec{E}_{outside} = \boxed{\frac{\rho R^3}{3\varepsilon_0 r^2} \hat{r}}$$

(b) $4\pi r^2 E_{in} = \frac{4\pi}{3} r^3 \rho / \varepsilon_0$

$$\vec{E}_{inside} = \boxed{\frac{\rho}{3\varepsilon_0} \vec{r}}$$

(c) $\rho' = -\rho$

$$\vec{E}' = \frac{-\rho}{3\varepsilon_0}(\vec{r} - \vec{a})$$

$$\vec{E}_{cavity} = \vec{E}_{inside} + \vec{E}' = \frac{\rho}{3\varepsilon_0}\vec{r} - \frac{\rho}{3\varepsilon_0}(\vec{r} - \vec{a}) = \boxed{\frac{\rho}{3\varepsilon_0}\vec{a}}$$

2. $\Phi = \sum_l \left(A_l r^l + B_l \frac{1}{r^{l+1}}\right) P_l(\cos\theta)$

inside : $\Phi < \infty \Rightarrow B_l^{in} = 0$

outside : $\Phi < \infty \Rightarrow A_l^{out} = 0$

Boundary conditions at $r = R$

$$\begin{cases} \phi^{in} = \phi^{out} \\ \frac{\partial \phi^{in}}{\partial r} - \frac{\partial \phi^{out}}{\partial r} = \frac{\sigma}{\varepsilon_0} \end{cases}$$

$$\begin{cases} \sum_l \left(A_l^{in} R^l - B_l^{out} \frac{1}{R^{l+1}}\right) P_l(\cos\theta) = 0 \\ \sum_l \left(A_l^{in} \cdot l R^{l-1} + B_l^{out} (l+1) \frac{1}{R^{l+2}}\right) P_l(\cos\theta) = \frac{\sigma_0}{\varepsilon_0} \cos\theta \end{cases}$$

By inspection: $\cos\theta = P_1(\cos\theta)$

Thus $A_l = B_l = 0$ for all $l \neq 1$, and

$$\begin{cases} A_1^{in} R - B_1^{out} R^{-2} = 0 \\ A_1^{in} + B_1^{out} \cdot \frac{2}{R^3} = \frac{\sigma_0}{\varepsilon_0} \end{cases}$$

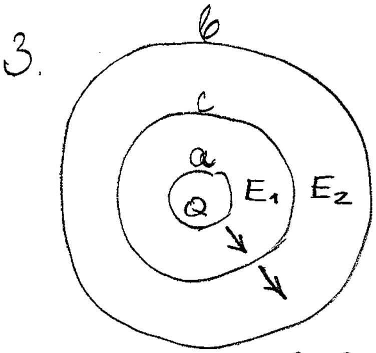
Thus $A_1^{in} = \frac{\sigma}{3\varepsilon_0}$ and $B_1^{out} = \frac{\sigma R^3}{3\varepsilon_0}$

(a) $\Phi^{in} = \boxed{\frac{\sigma}{3\varepsilon_0} r \cos\theta}$

(b) $\vec{E}^{in} = -\vec{\nabla}\Phi = \boxed{-\left(0, 0, \frac{\sigma}{3\varepsilon_0}\right)}$

$\Phi^{out} = \boxed{\frac{\sigma R^3}{3\varepsilon_0 r^2} \cos\theta}$

3.



Assuming $Q>0$

(a) Gauss law:

$$D = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$E_1 = \frac{D}{\varepsilon_1} = \frac{Q}{4\pi\varepsilon_0\varepsilon_1 r^2}$$

$$E_2 = \frac{D}{\varepsilon_2} = \frac{Q}{4\pi\varepsilon_0\varepsilon_2 r^2}$$

(b)

$$\Phi_a - \Phi_c = \int_a^c dr\, E_1 = \frac{Q}{4\pi\varepsilon_0\varepsilon_1}\left(\frac{1}{a} - \frac{1}{c}\right)$$

$$\Phi_b - \Phi_c = \int_c^b dr\, E_2 = \frac{Q}{4\pi\varepsilon_0\varepsilon_2}\left(\frac{1}{c} - \frac{1}{b}\right)$$

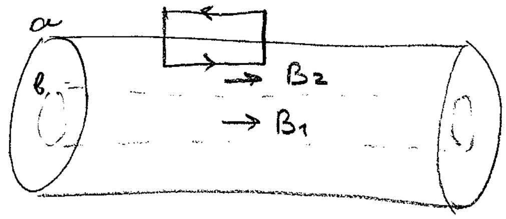
$$\Phi_a - \Phi_b = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{\varepsilon_1}\left(\frac{1}{a} - \frac{1}{c}\right) + \frac{1}{\varepsilon_2}\left(\frac{1}{c} - \frac{1}{b}\right)\right]$$

$$C = \frac{Q}{\Phi_a - \Phi_b} = 4\pi\varepsilon_0\left[\frac{1}{\varepsilon_1}\left(\frac{1}{a} - \frac{1}{c}\right) + \frac{1}{\varepsilon_2}\left(\frac{1}{c} - \frac{1}{b}\right)\right]^{-1}$$ $\checkmark$

(c) Gauss law:

$$\sigma = \varepsilon_0(E_2 - E_1)\Big|_{r=c} = \frac{Q}{4\pi\varepsilon_0 c^2}\left(\frac{1}{\varepsilon_2} - \frac{1}{\varepsilon_1}\right)$$ $\checkmark$

4.



(a) Ampere's law:

$$lH = Inl$$

$$H = In$$

$$B_1 = \mu_1\mu_0 H = \mu_1\mu_0 In \quad ; r<b$$

$$B_2 = \mu_2\mu_0 H = \mu_2\mu_0 In \quad ; b<r<a$$

✓

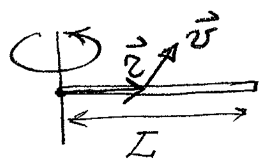
(b) Flux:

$$\Phi = N\cdot\left(B_1\cdot\pi b^2 + B_2\pi(a^2-b^2)\right) \qquad N = n\cdot l$$

$$= N\mu_0 In\cdot\pi\left(\mu_1 b^2 + \mu_2(a^2-b^2)\right)$$

$$L = \frac{\Phi}{I} = \mu_0 n^2\pi\left(\mu_1 b^2 + \mu_2(a^2-b^2)\right)l$$ ✓

5.



(a) $$m = \frac{1}{2}\int_0^L dr\, r v \lambda = \frac{\lambda\omega}{2}\int_0^L dr\, r^2 = \frac{\lambda\omega L^3}{6}$$

$$\boxed{\vec{m} = \frac{\lambda\omega L^3}{6}\cdot\hat{z}}$$

(b) $$p = \int_0^L dr\,\lambda r = \frac{\lambda L^2}{2}$$

$$\boxed{\vec{p} = \frac{\lambda L^2}{2}(\cos\omega t, \sin\omega t, 0)}$$

(c) $$\vec{p} = \mathrm{Re}\,\underbrace{\frac{\lambda L^2}{2}}_{\vec{p}_\omega} e^{i\omega t}$$

$$\frac{dE}{dt} = \frac{|\vec{p}_\omega|^2\omega^4}{12\pi\varepsilon_0 c^3} = \boxed{\frac{\lambda^2 L^4\omega^4}{48\pi\varepsilon_0 c^3}}$$