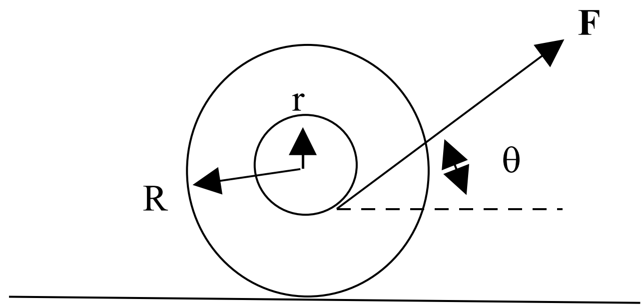
University of Illinois at Chicago
Department of Physics

Classical Mechanics
Qualifying Examination

January 3, 2006
9:00 am-12:00 pm

Full credit can be achieved from completely correct answers to 4 questions. If the student attempts all 5 questions, all of the answers will be graded, and the top 4 scores will be counted toward the exam's total score

1. A toy consists of two concentric cylinders with inner radius r and outer radius R. A string is wound around the inner radius and the outer radius can roll without slipping on a rough floor. The string is pulled at angle θ with respect to the horizontal.



a. Draw the free body diagram.
b. Calculate the angular acceleration.
c. Prove that there exists a critical angle θ_c, where if $\theta < \theta_c$ the cylinder rolls away from the direction it is pulled, and if $\theta > \theta_c$ the cylinder rolls toward the direction it is pulled.
d. Determine θ_c

2. A positron e+ with energy of 250 GeV/c^2 travels along the x axis and collides with a stationary electron. A single particle V is produced and only V remains after the collision. Later, V decays into two identical mass (m= 0.1 GeV/c^2), unstable muons μ^+ and μ^- which have lifetimes of 2×10^{-6} s in their rest frame.

a. Calculate the v/c of the positron.
b. What is the mass of particle V?
c. What is the total energy of the particle V in its rest frame?
d. What are the momenta of the electron and positron in the V rest frame?
e. If the muon decays perpendicularly to the x axis in the V rest frame, what approximate angle does it make with respect to the x axis in the lab frame?
f. How far would the muon travel in one lifetime as measured in the lab frame?

3. A particle of mass m moves in a field $F = f(r)r$, where $f(r) = -\frac{C}{r^3}$ and C>0.

a. Calculate $\frac{dl}{dt}$, where $l = mr^2 \frac{d\theta}{dt}$.

b. Derive the equation of motion for r and show you can write it in form

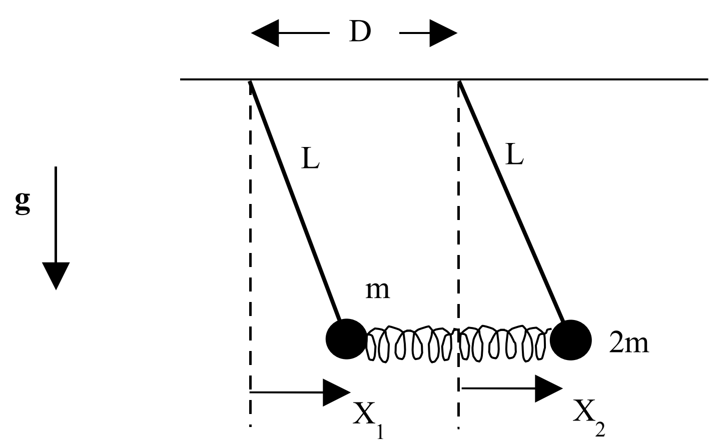
$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{l^2u^2} f(\frac{1}{u}), \text{ where } u = \frac{1}{r}.$$

Hint. Find the relationship of $\frac{d}{d\vartheta}$ to $\frac{d}{dt}$ for the central force.

c. Show that a possible solution is spiral orbit of the form $r = r_0 e^{\beta\theta}$. Find all possible solutions.

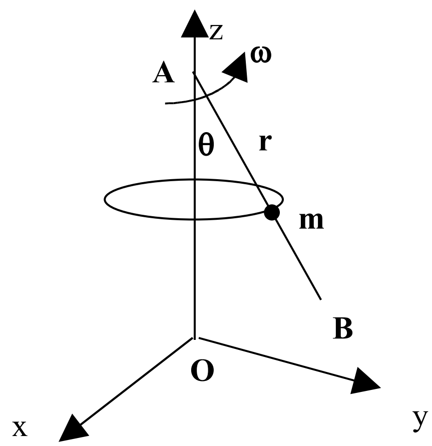
d. Show that θ varies logarithmically with t for the spiral orbit from part c.
Hint: integrate *l* to find θ(t).

4. Two pendulums are coupled by a massless spring with spring constant k. Both pendulums have massless springs of length L. They are separated by distance D. The masses are m and 2m. Consider small oscillations.



a. Solve for the normal modes of the pendulums.
b. Determine the normal coordinates that undergo simple harmonic motion.

5..A bead of mass m moves along a frictionless wire AB. The wire is fixed at point A and rotates with angular frequency ω about the z axis. θ is fixed



a. Determine the Lagrangian in terms of r, θ and azimuthal angle
b. Determine the Lagrange equation as a function of m, $\frac{dr}{dt}$, ω, r and θ.
c. Solve the equation of motion.