1 A) FIND ELECTRIC FIELD DUE TO EACH OBJECT:

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(r) \cdot 4\pi r^2 = \frac{1}{\varepsilon_0} \cdot \frac{4}{3}\pi r^3 \rho_0 \, , \quad r < a$$

$$\vec{E}_{SPHERE} = \begin{cases} \frac{\rho_0 r}{3\varepsilon_0} \hat{r}, & r < a \\ \frac{\rho_0 a^3}{3\varepsilon_0 r^2} \hat{r}, & r > a \end{cases}$$

$$\vec{E}_{PLANE} = \pm \frac{\sigma_0}{2\varepsilon_0} \hat{z}$$

$$V(d) - V(0) = -\int_0^d \vec{E} \cdot d\vec{l}$$

$$= -\int_0^a \left(\frac{\rho_0 r}{3\varepsilon_0} - \frac{\sigma_0}{2\varepsilon_0} \right) dr - \int_a^d \left(\frac{\rho_0 a^3}{3\varepsilon_0 r^2} - \frac{\sigma_0}{2\varepsilon_0} \right) dr$$

$$= -\frac{\rho_0}{3\varepsilon_0} \cdot \frac{1}{2} a^2 + \frac{\sigma_0}{2\varepsilon_0} a + \frac{\rho_0 a^3}{3\varepsilon_0} \left(\frac{1}{d} - \frac{1}{a} \right) + \frac{\sigma_0}{2\varepsilon_0} (d - a)$$

$$= \frac{\rho_0 a^2}{3\varepsilon_0} \left(-\frac{1}{2} - 1 \right) + \frac{\rho_0 a^3}{3\varepsilon_0 d} + \frac{\sigma_0 d}{2\varepsilon_0}$$

$$= -\frac{\rho_0 a^2}{2\varepsilon_0} \quad + \quad '' \quad ''$$

$$= \frac{\rho_0 a^2}{\varepsilon_0} \left(-\frac{1}{2} + \frac{a}{3d} \right) + \frac{\sigma_0 d}{2\varepsilon_0}$$

B) NET FORCE IS ONLY DUE TO THE PLANE

$$\vec{F} = q\vec{E}_{PLANE} = \frac{4}{3}\pi a^3 \rho_0 \cdot \frac{\sigma_0}{2\varepsilon_0} \hat{z}$$

(2) A) CONSIDER THE BOUND SURFACE CHARGE AROUND THE PERIMETER OF THE DISC.

$$\sigma_b = \vec{P} \cdot \hat{s} = P\cos\varphi$$

SINCE THE DISC IS THIN, TREAT AS A CIRCULAR LINE CHARGE WITH DENSITY $\lambda(\varphi) = Pd\cos\varphi$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\hat{\mathscr{r}}\,\lambda\, dl'}{\mathscr{r}^2} = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi} \frac{\hat{\mathscr{r}}\, Pd\cos\varphi' R\, d\varphi'}{\mathscr{r}^2}$$

WHERE $\vec{\mathscr{r}} = z\hat{z} - R\cos\varphi'\hat{x} - R\sin\varphi'\hat{y}$

$$|\vec{\mathscr{r}}| = \sqrt{z^2+R^2}$$

ALL BUT x COMPONENT WILL INTEGRATE TO ZERO

$$\vec{E} = -\frac{PdR^2}{4\pi\varepsilon_0}\int_0^{2\pi} \frac{\cos^2\varphi'\, d\varphi'\ \hat{x}}{(z^2+R^2)^{3/2}}$$

$$= -\frac{PdR^2}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}}(\pi)\ \hat{x}$$

B) ~~WHEN $r \gg R$ LOOKS LIKE POINT-LIKE DIPOLE~~

~~$\Rightarrow \vec{E} \approx \frac{1}{4\pi\varepsilon_0}$~~

AT ORIGIN $(z=0)$ $\vec{E} = -\frac{PdR^2}{4\varepsilon_0 R^3}\hat{x} = -\frac{\vec{P}d}{4\varepsilon_0 R}$

THIS IS THE "MACROSCOPIC FIELD" WHICH IS THE AVERAGE FIELD OVER A VOLUME CONTAINING MANY ATOMS. THE ACTUAL, MICROSCOPIC FIELD IS MUCH DIFFERENT, PARTICULARLY NEAR ATOM NUCLEI.

C) WHEN $r \gg R$ LOOKS LIKE POINT-LIKE DIPOLE

$$\rightarrow \vec{E} \approx \frac{1}{4\pi\varepsilon_0}\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{3(\cancel{p\hat{x}\cdot z\hat{z}})\hat{z} - p\hat{x}}{z^3}$$

WHERE $\vec{p}$ = TOTAL DIPOLE MOMENT = $\vec{P}\cdot\pi R^2 d$

$$\vec{E} \approx -\frac{PR^2d}{4\varepsilon_0 z^3}\hat{x} \quad \text{(MATCHES PART A IN LIMIT } z \gg R)$$

(3) A) RC CIRCUIT: $I_1(t) = I_0 e^{-t/RC}$

NOTE THAT AT $t=0$, NO VOLTAGE ON C, SO $I_0 = \frac{V_0}{R}$

$$I_1(t) = \frac{V_0}{R} e^{-t/RC}$$

B) LONG STRAIGHT WIRE PRODUCES A MAGNETIC FIELD:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc.}$$

$$B \cdot 2\pi s = \mu_0 I$$

$$\vec{B} = \frac{\mu_0 I}{2\pi s} \hat{\varphi}$$

FLUX THROUGH ONE TURN OF COIL:

$$\Phi_1 = \iint \vec{B} \cdot d\vec{a} = h \cdot \int_a^b \frac{\mu_0 I}{2\pi s} ds$$

$$= \frac{\mu_0 h I}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$\Phi_N = N \cdot \Phi_1$$

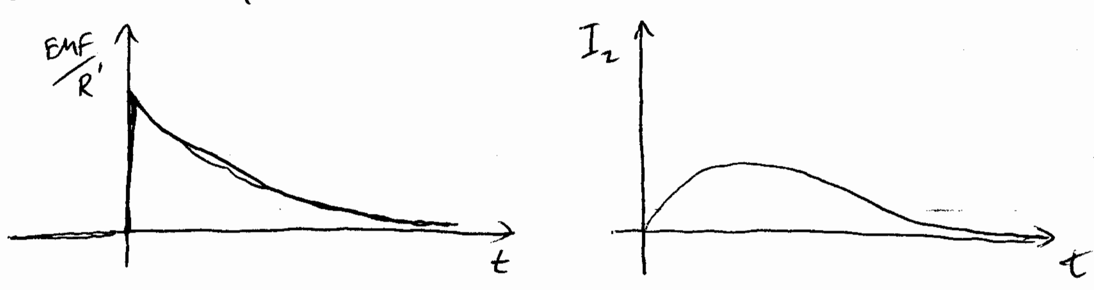
$$EMF = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left(\frac{N\mu_0 h V_0}{2\pi R} \ln\left(\frac{b}{a}\right) e^{-t/RC}\right)$$

$$= \frac{N\mu_0 h V_0}{2\pi R^2 C} \ln\left(\frac{b}{a}\right) e^{-t/RC}$$

CURRENT IN R' FLOWS FROM BOTTOM TO TOP FOR $t>0$

C) THE CURRENT CANNOT CHANGE QUICKLY DUE TO THE (SELF) INDUCTANCE OF THE COIL. THE SELF-EMF IN THE COIL WILL OPPOSE THE EMF INDUCED BY THE STRAIGHT WIRE. IN PARTICULAR, THE CURRENT WILL NOT RISE ABRUPTLY AS IT DOES IN CIRCUIT 1.

QUALITATIVELY:



(4) A) EFFECT OF MAGNETIZED BODY TREATED USING BOUND CURRENT ON THE CYLINDER SIDES

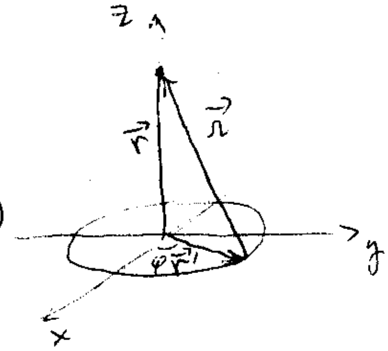
$$\vec{K}_b = \vec{M} \times \hat{n} = M\hat{z} \times \hat{s} = M\hat{\varphi}$$

SINCE WE ASSUME THE DISC IS THIN, CAN BE TREATED AS A CURRENT LOOP OF RADIUS R AND $I = Kd = \left(\frac{\chi}{\mu_0}\right) d B_0 \cos(\omega t)$

MAGNETOSTATIC LIMIT REQUIRES THAT THE WAVELENGTH OF LIGHT IS MUCH GREATER THAT THE SIZE OF THE OBJECT AND ALSO MUCH GREATER THAN THE DISTANCE TO THE OBSERVATION POINT:

$$\lambda \gg R \quad \text{AND} \quad \lambda \gg r$$

$$\Rightarrow \frac{2\pi c}{\omega} \gg R \quad \text{AND} \quad \frac{2\pi c}{\omega} \gg r$$



BIOT-SAVART: $\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}' \times \hat{\imath}}{\imath^2}$

$$\vec{\imath} = \vec{r} - \vec{r}' = z\hat{z} - (R\cos\varphi'\hat{x} + R\sin\varphi'\hat{y})$$

$$d\vec{l}' = R\,d\varphi'\hat{\varphi} = R\,d\varphi'(-\sin\varphi'\hat{x} + \cos\varphi'\hat{y})$$

$$d\vec{l}' \times \vec{\imath} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -R\sin\varphi' d\varphi' & +R\cos\varphi' d\varphi' & 0 \\ -R\cos\varphi' & -R\sin\varphi' & z \end{vmatrix}$$

NEGLECT $\hat{x}, \hat{y}$ COMPONENTS SINCE THEY INTEGRATE TO ZERO

$$B_z = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{R^2\sin^2\varphi' - (-R^2\cos^2\varphi')}{(R^2+z^2)^{3/2}}\, d\varphi' = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}$$

$$\vec{B}(t) = \hat{z}\,\frac{\mu_0 R^2}{2(R^2+z^2)^{3/2}} \left(\frac{\chi}{\mu_0}\right) dB_0 \cos(\omega t)$$

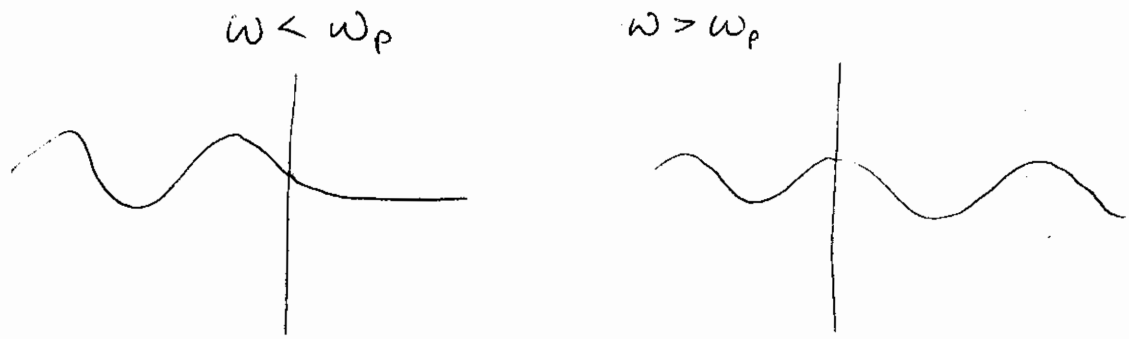
(4) B) NON - MAGNETOSTATIC

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{I(\vec{r}', t - \frac{r}{c})\, \hat{\varphi}}{r}\, dl'$$

$$= \frac{\mu_0}{4\pi} \int_0^{2\pi} \left(\frac{\chi}{\mu_0}\right) \frac{dB_0 \cos\left(\omega\left(t - \frac{r}{c}\right)\right) \hat{\varphi}}{r} R\, d\varphi'$$

$$r = |\vec{r} - \vec{r}'| = \sqrt{(x - R\cos\varphi')^2 + (y - R\sin\varphi')^2 + z^2}$$

⑤ A) FOR $\omega < \omega_p$, k BECOMES IMAGINARY, SO WAVE IS EXPONENTIALLY DAMPED IN THE PLASMA. FOR $\omega > \omega_p$ THE WAVE TRAVELS THROUGH THE PLASMA. IN THIS CASE, SINCE $k = \sqrt{\frac{\omega^2 - \omega_p^2}{c^2}}$ THE MAGNITUDE OF k IS SMALLER THAN IN VACUUM $\left(k_{VAC} = \frac{\omega}{c} \right)$ SO THE WAVELENGTH IS LONGER.



B) $$\vec{E} = \begin{cases} E_I e^{i(k_1 z - \omega t)} \hat{x} + E_R e^{i(-k_1 z - \omega t)} \hat{x}, & z < 0 \\ E_T e^{i(\tilde{k}_2 z - \omega t)} \hat{x}, & z > 0 \end{cases}$$

$$\vec{B} = \begin{cases} \frac{E_I}{c} e^{i(k_1 z - \omega t)} \hat{y} - \frac{E_R}{c} e^{i(-k_1 z - \omega t)} \hat{y}, & z < 0 \\ E_T \frac{\tilde{k}_2}{\omega} e^{i(\tilde{k}_2 z - \omega t)} \hat{y}, & z > 0 \end{cases}$$

BOUNDARY COND.: $E_{\parallel}$ CONTINUOUS: $E_I + E_R = E_T$

$H_{\parallel}$ CONTINUOUS: $\frac{E_I}{\mu_0 c} - \frac{E_R}{\mu_0 c} = E_T \frac{\tilde{k}_2}{\mu_0 \omega}$

SOLVE SYSTEM OF EQUATIONS

$$\Rightarrow \quad E_T = \frac{2}{1 + \frac{\tilde{k}_2 c}{\omega}} E_I \quad , \quad E_R = \frac{1 - \frac{\tilde{k}_2 c}{\omega}}{1 + \frac{\tilde{k}_2 c}{\omega}} E_I$$

c) SOLVE FOR $\tilde{k}_2$ FROM DISPERSION RELATION

$$\frac{3}{4} \omega_p^2 = \omega_p^2 + c^2 k_2^2$$

$$\tilde{k}_2 = i \frac{\omega_p}{2c}$$

(5) c) $E_T = \dfrac{2}{1+\frac{i}{\sqrt{3}}} E_I$

$$B_T = \frac{\tilde{k}_2}{\omega} E_T = \frac{i}{\sqrt{3}c} E_T = \frac{2i}{\sqrt{3}c\left(1+\frac{i}{\sqrt{3}}\right)} E_I$$

NOW EXPRESS IN POLAR FORM

$$B_T = \frac{2i}{\sqrt{3}c\left(1+\frac{i}{\sqrt{3}}\right)} \cdot \frac{\left(1-\frac{i}{\sqrt{3}}\right)}{\left(1-\frac{i}{\sqrt{3}}\right)} E_I = \frac{2\left(i+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}c\left(1+\frac{1}{3}\right)} E_I$$

$$= \left(\frac{1}{2c} + \frac{\sqrt{3}}{2c} i\right) E_I = \frac{1}{c}\sqrt{\frac{1}{4}+\frac{3}{4}}\, e^{i\theta} E_I$$

$$= \frac{1}{c} e^{i\theta} E_I$$

WHERE $\theta = \arctan(\sqrt{3}) = \frac{\pi}{3}$ OR $60°$

$$B_T = \frac{1}{c} e^{i\frac{\pi}{3}} E_I$$