University of Illinois at Chicago
Department of Physics

Classical Mechanics
Qualifying Examination

January 4, 2006
9.00 am – 12:00 pm

Full credit can be achieved from completely correct answers to <u>4 questions</u>. If the student attempts all 5 questions, all of the answers will be graded, and the <u>top 4 scores</u> will be counted toward the exam's total score.

Problem 1

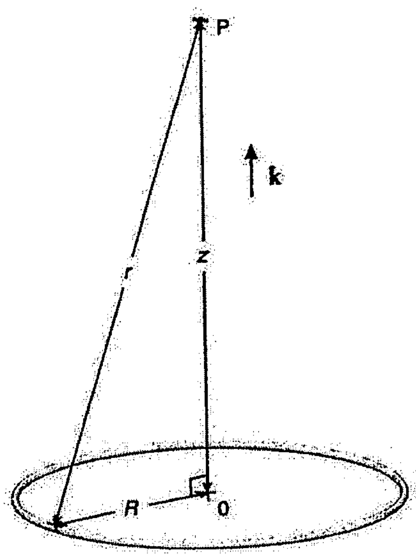
A uniform circular ring of radius R and total mass M lies on the $x-y$ plane with its center at the origin.

a) Find the gravitational force F_G exerted on a particle of mass m located at a distance z along the z-axis.

b) Find the potential energy of the particle as a function of z assuming that $V \to 0$ when $z \to \infty$.

c) Find the value of z for which $|F_G|$ is a maximum, and calculate $|F_G|$ at that point.

d) Show that for $z << R$ the motion of the particle is harmonic with time, and find the frequency of the oscillation.



Problem 2

A projectile is launched at an angle of 45 degrees with an initial kinetic energy E_0. At the top of its trajectory, the projectile explodes into two fragments. The explosion imparts an additional mechanical energy E_0 to the system. One fragment of mass m_1 travels straight down with an unknown velocity v_1. Assume the motion is in the x-y plane.

a) Find the components of the velocity v_{2x} and v_{2y} of the second fragment of mass m_2, and the magnitude of the velocity v_1 of the first fragment of mass m_1.

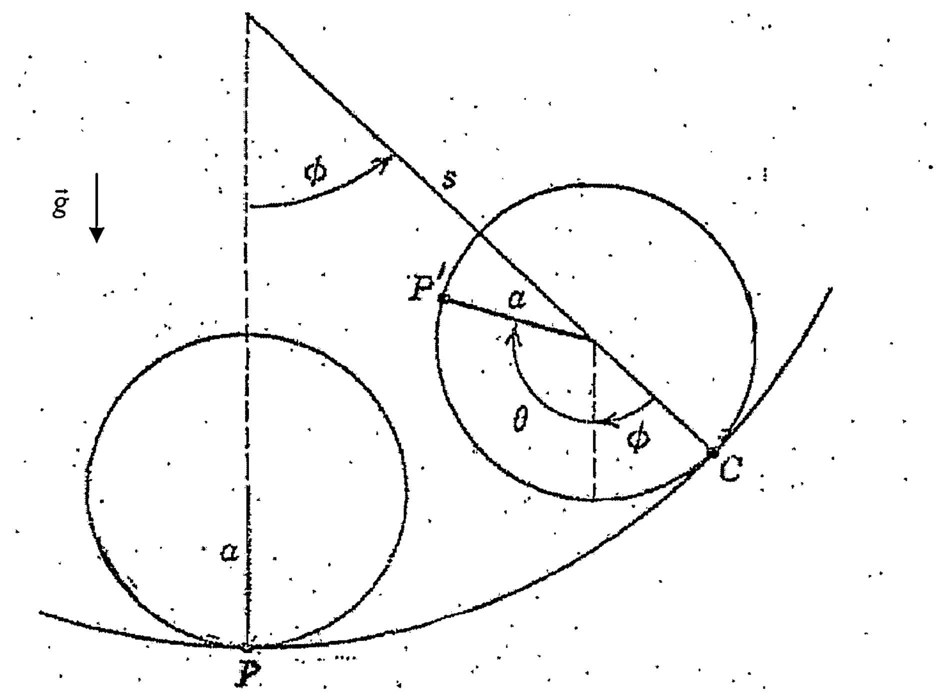
b) What is the ratio of masses $\frac{m_1}{m_2}$ that maximizes m_1?

c) Find the horizontal range for m_2 measured from the initial launch position of the projectile if $m_1 = 2kg$, $m_2 = 3kg$, $E_0 = 100J$. Consider $g = 10m/s^2$.

Problem 3

A uniform ball bearing of radius a, mass M, and moment of inertia around its center of mass $I = \frac{2}{5}Ma^2$ rolls back and forth without slipping on a cylindrical track of radius s. The motion is constrained to the plane of the paper, and a uniform gravitational field of strength g is present, as shown in the figure. The angle ϕ with vertex at the center of the circle of radius s measures the position of the center of mass of the sphere. The sphere rotates through an angle θ when the center of mass moves through an angle ϕ. During this motion, the point P moves to P'.

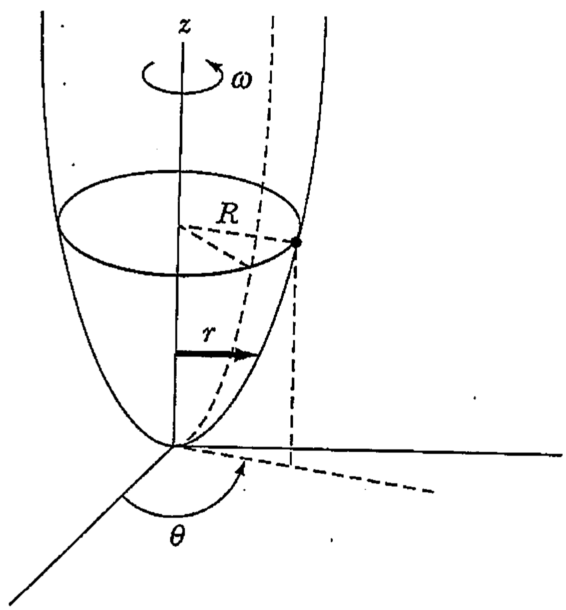
a) Find the equation of motion for the angle ϕ.
b) Find the frequency of the oscillation for small amplitudes
c) Calculate the frequency of small oscillations in the limit $a << s$. Does this frequency equal the one for a pendulum of mass M and length s? Explain.



Problem 4

A bead slides frictionless along a wire bent in the shape of a parabola $z = cr^2$. The wire is rotating about its vertical symmetry axis with angular velocity ω.
Choose r, θ, and z as the generalized coordinates for the problem.

a) Find the kinetic energy of the bead.
b) Find the potential energy of the bead choosing $U = 0$ at $z = 0$.
c) Write the equations of constraint for the system. How many degrees of freedom does the system have?
d) Find Lagrange's equations of motion for the bead.
e) Find the value of c that causes the bead to rotate in a circle of fixed radius.



<u>Problem 5</u>

A wheel travels with constant speed V_0 around a circular track of radius ρ. A rod connects the center of the wheel with the center of curvature C of the track. Let b denote the radius of the bicycle wheel. Choose a coordinate system with origin at the center of the wheel and with the horizontal x' axis pointing towards the center of curvature C of the track. The z' axis remains vertical as shown in the Figure.

a) Find the acceleration of O' as it rotates about point C.

b) Find the acceleration for the point at the top of the wheel with respect to O'.

c) Find the Coriolis, Tangential and Centripetal acceleration for the point at the top of the wheel with respect to O'.

d) Find the net acceleration, relative to the ground, of the highest point of the wheel.