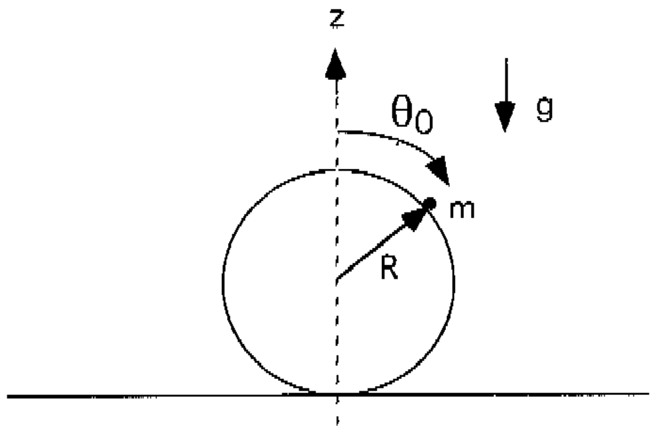
University of Illinois at Chicago
Department of Physics

Classical Mechanics
Preliminary Examination

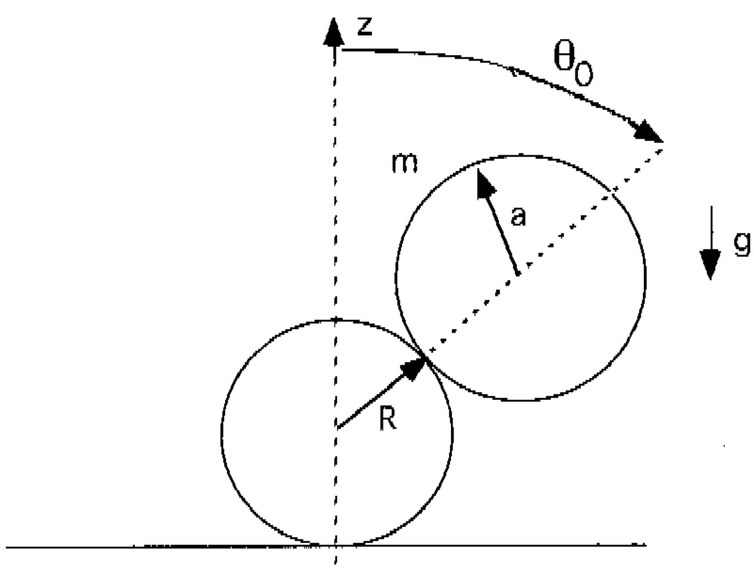
January 4, 2005
9.00 am – 12:00 pm

Full credit can be achieved from completely correct answers to 4 questions. If the student attempts all 5 questions, all of the answers will be graded, and the top 4 scores will be counted toward the exam's total score.

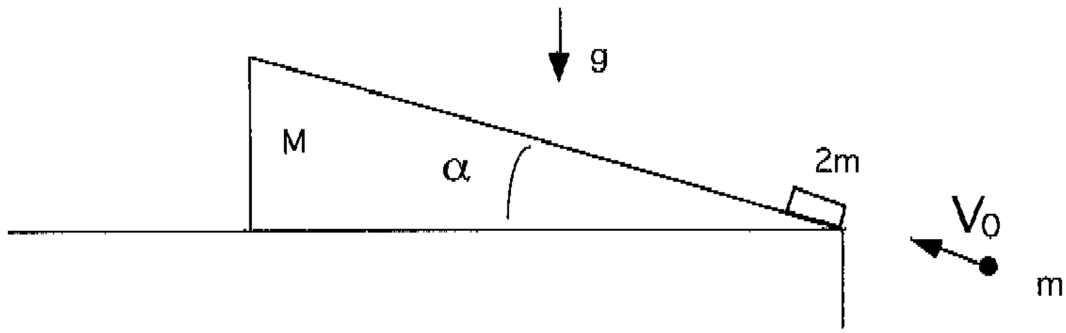
1. (a) At $t = 0$, a point particle of mass m is at rest on the frictionless surface of a fixed sphere. The sphere has radius R, and the particle is initially at the angle θ_0 relative to the vertical z-axis through the center of the sphere. Gravity is directed as shown. The particle is released. It leaves the surface of the sphere at angle θ_1. Find θ_1 in terms of θ_0.



(b) The point particle of the above problem is replaced by a ball of mass m and radius a. The moment of inertia of the ball about its axis is $2ma^2/5$. The bottom sphere is still fixed and cannot move, but there is now friction so the ball starts at $t = 0$ at the angle θ_0 and rolls without slipping until it leaves the surface of the sphere at angle θ_1. Find θ_1 in terms of θ_0.



2. A slider of mass 2m is initially at rest at the bottom edge of a wedge of mass M and angle a. The wedge has a frictionless surface and rests on a frictionless table. At t = 0 a bullet of mass m and velocity V_0 travelling parallel to the upper surface of the wedge hits and sticks in the slider.



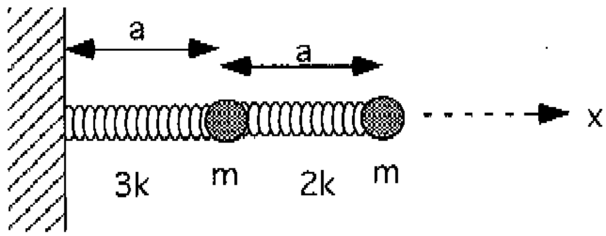
(a) What is the maximum height above the table reached by the slider?

(b) At that time, what is the wedge's velocity?

(c) At what time does it reach its maximum height?

(d) How far has the wedge moved from the table edge at that time?

3. 2 objects, each of mass m, are attached to each other by a spring, and the left mass is also attached by a spring to a fixed wall. The springs are of equilibrium length a. The figure shows a top view. The masses are on a frictionless surface, and can only move along the x-axis. The left spring has spring constant 3k, and the right spring has spring constant 2k.



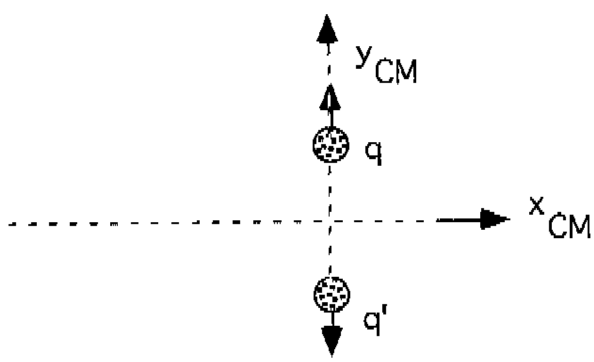
(a) Find the Lagrangian for this system.

(b) Find the normal modes and their frequencies.

(c) At t = 0 the left hand mass is displaced from its equilibrium position the distance - b to the left, and the right hand mass is displaced from its equilibrium position the distance + b to the right. The masses are then released with zero initial velocity. Find the positions of both masses as functions of t.

4. A particle of mass m moves along the x axis with relativistic momentum p_0. It collides with another mass m particle that is stationary.

A reaction occurs, with the two mass m particles turning into two new particles, one of mass M_1, and the other of mass M_2, with $M_1 > m$ and also $M_2 > m$. After the collision, the two new particles move in opposite directions along the center of momentum frame y-axis. The mass M_1 particle moves along the positive y_{CM} direction has momentum $\vec{q}$, the mass M_2 particle has momentum $\vec{q}'$. The lab frame x axis and the CM frame x-axis are parallel.



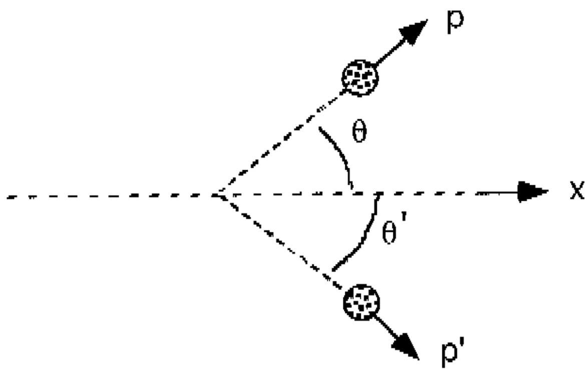
(a) How is $\vec{q}$ related to $\vec{q}'$?

(b) Find the minimum initial momentum $p_0 = p_{min}$ required for this reaction to occur.

If $p_0 > p_{min}$, find, in terms of p_0, m, M_1, M_2 ,

(c) The momentum magnitudes, q, q' .

In the lab frame, the final particles have momenta $\vec{p}$ and $\vec{p}'$, making the angles θ and θ' with the x-axis.



In the following, take $M_1 = M_2 = M$, and find in the lab frame;

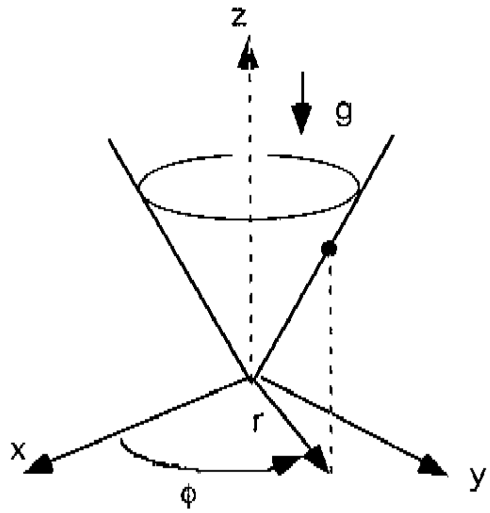
(d) the momentum magnitudes p, p' ,

(e) the angles θ, θ' .

5. A particle is confined to, and moving on, a frictionless surface of revolution, a cone where

$$z = ar \quad ,$$

for some positive constant a.



(a) Find the Lagrangian, in terms of r and ϕ.

(b) Find the Hamiltonian,

(c) Find the equations of motion.

(d) Determine if it is possible to have on this surface circular orbits about the z axis.

(e) If such an orbit exists, find its radius r_0 in terms of g, a and the orbital angular frequency ω.

(f) If such an orbit exists, let $r = r_0 + \delta r$, where $\delta r << r_0$. Expand the equation for $\ddot{r}$ keeping only terms of first order in δr and its time derivatives, and use this first order equation to find the angular frequency of oscillation across the orbit, ω_{osc}, in terms of a and ω.