University of Illinois at Chicago
Department of Physics

Electricity & Magnetism
Preliminary Examination

January 3, 2005
9.00 am – 12:00 pm

Full credit can be achieved from completely correct answers to 4 questions. If the student attempts all 5 questions, all of the answers will be graded, and the top 4 scores will be counted toward the exam's total score.

Miscellaneous Equations:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots$$

$$\ln(1+x) = x - \tfrac{1}{2}x^2 + \cdots$$

$$e^x = 1 + x + \tfrac{1}{2}x^2 + \cdots$$

$$\sin(x) = x - \tfrac{1}{3!}x^3 + \cdots$$

$$\cos(x) = 1 - \tfrac{1}{2}x^2 + \cdots$$

$$P_0(x) = 1$$

$$P_1(x) = x$$

$$P_2(x) = (3x^2 - 1)/2$$

$$P_3(x) = (5x^3 - 3x)/2$$

$$\nabla \cdot \mathbf{E} = \frac{1}{\varepsilon_0}\rho$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \cdot \mathbf{D} = \rho_f$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$$

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/Nm^2}$$

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2}$$

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

$$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}$$

$$\mathbf{D} = \varepsilon \mathbf{E}$$

$$\mathbf{H} = \frac{1}{\mu}\mathbf{B}$$

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

$$\oint \vec{A} \cdot d\vec{\ell} = \Phi_B$$

$$\mathcal{E} = \oint \vec{f} \cdot d\vec{\ell} = \oint (\vec{E} + \vec{v} \times \vec{B}) \cdot d\vec{\ell}$$

$$\mathcal{E} = -\tfrac{d\Phi}{dt},\ \mathcal{E} = -L\tfrac{dI}{dt}$$

$$\vec{E} = -\vec{\nabla} V - \tfrac{\partial \vec{A}}{\partial t},\ \vec{B} = \vec{\nabla} \times \vec{A}$$

$$C \equiv \frac{Q}{V},\ U = \tfrac{1}{2}CV^2,\ L \equiv \Phi_B / I,\ U = \tfrac{1}{2}LI^2$$

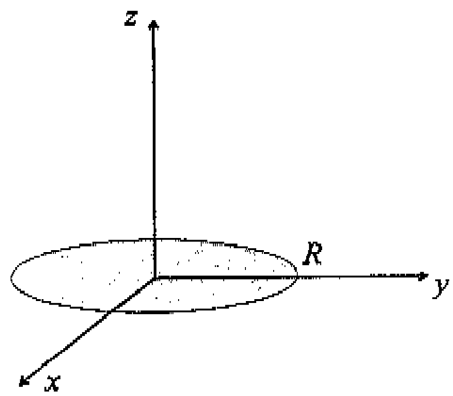
$$u_{em} = \tfrac{1}{2}(\vec{E} \cdot \vec{D} + \vec{B} \cdot \vec{H})$$

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\frac{dW}{dt} = -\frac{dU_{em}}{dt} - \oint \vec{S} \cdot d\vec{a}$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

1. A disc of radius R and uniform, surface charge density σ, is located in the xy-plane, centered at the origin as shown in the figure below.

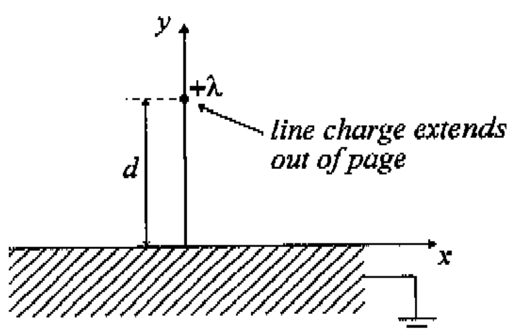


a) Compute the electric potential for a point along the z-axis, V(z).

b) Show that your result from part (a) has the proper leading order dependence on z as $z \to \infty$.

c) Write an approximate expression for the potential $V(r,\theta)$ far away from the disc, with terms up to order r^{-3}.

2. An infinite, grounded, conducting plane is in the *xz*-plane. A line of charge, with linear charge density λ runs parallel to the *z*-axis, a distance d above the conducting plane. Assume the whole region exclusive of the conductors is vacuum.

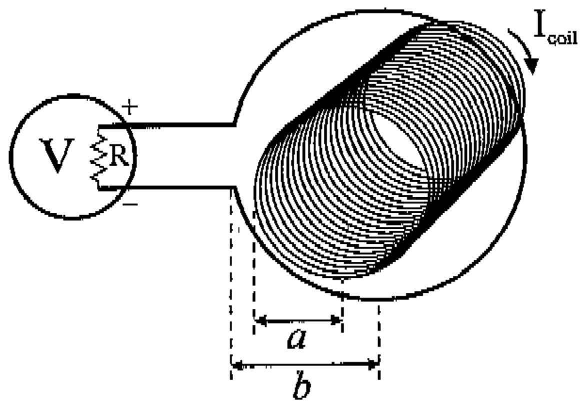


a) Compute the electric potential V(x,y,z) for y > 0.

b) Compute the capacitance per unit length of a thin wire of radius a, placed a distance d above a grounded plane. Assume that the wire radius is much smaller than d (i.e. $a \ll d$) so that the solution of part (a) is approximately correct in the region exclusive of the conductors. Also write a numerical value in units of F/m, where $d = 0.10$ m, $a = 0.001$ m.

c) Compute the force per unit length on the wire (including the direction).

3. Consider a long, cylindrical coil, of radius a, length ℓ, and N turns. The coil is located at the center of a circular, wire loop of radius b. This loop has a voltmeter, having internal resistance R in series with it, as shown below. The self-inductance of the coil is L_{coil} and the self-inductance of the loop is L_{loop}.

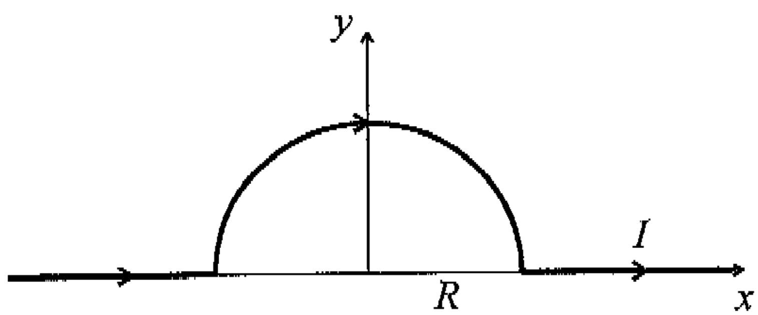


(a) Why does it simplify the analysis of induced voltages to have the coil be "long"? In particular, why should $\ell >> b$?

(b) Assuming the voltmeter is ideal, determine the magnitude and sign of the voltage that is measured as the current (flowing in the direction shown in the figure) is steadily increased from zero to I_F in a time τ [take note of the polarity of the voltmeter on the figure].

(c) In the case where the voltmeter is not ideal, re-answer part (b). Comment qualitatively on the time dependence of the voltmeter reading.

4. A semicircular wire of radius R is centered at the origin, while straight segments extend to infinity along the x-axis, as shown below. A uniform current I_0 is suddenly turned on at t=0, remaining constant thereafter.

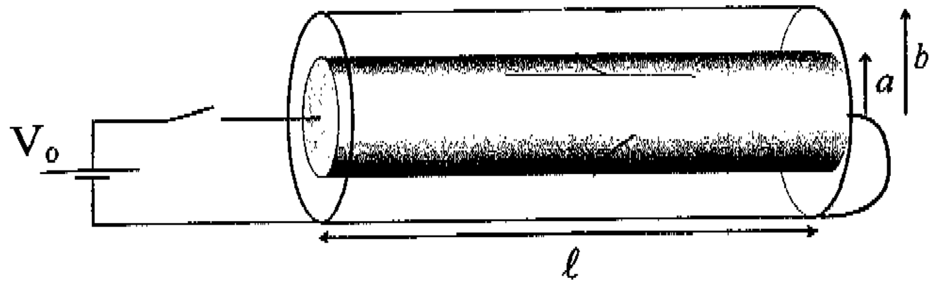


(a) Find the vector ($\vec{A}$) and scalar (V) potentials vs. time at the origin.

(b) Using the result of part (a), find the electric field vs. time at the origin, or briefly discuss how it could be found with more information.

(c) Using the result of part (a), find the magnetic field vs. time at the origin, or briefly discuss how it could be found with more information.

5. Consider a long, coaxial cable of radius b and length ℓ, with a center conductor of radius a. The center conductor is made of material having resistivity ρ and linear magnetic permeability μ. The outer shield is a perfect conductor and is shorted to the inner conductor at the right end. At $t = 0$ a voltage V_0 is suddenly applied at the left end and remains constant thereafter. Assuming the current is uniform along the length of the cable and that $\ell >> b$, determine the current as a function of time I(t).



μ

1) Energy

2) $I(t)$