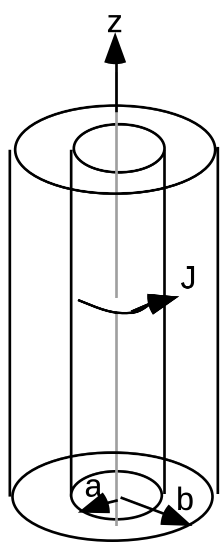
ELECTROMAGNETISM PRELIMINARY EXAM
January 2004



1. The z-axis coincides with the axis of an infinite conducting cylinder of inner radius a, outer radius b. For a < r < b, there is a current density flowing around the wire, $\vec{\mathbf{J}} = J_0 \hat{\phi} \quad A/m^2$.

For the three regions

(I) $0 < r < a$,

(II) $a < r < b$

(III) $b < r$,

Find:

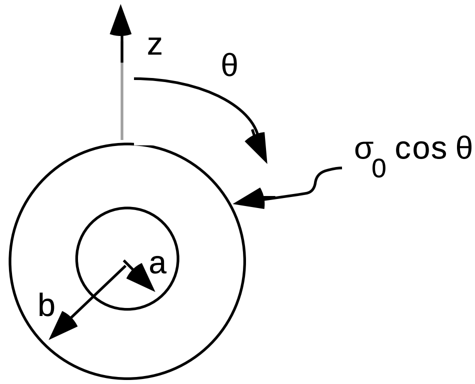
(a) The **B** field .

(b) The **A** field .

And then find

(c) The pressure exerted by the magnetic field on the cylinder.

2. A grounded conducting sphere of radius a is inside a sphere of radius b. The region between radius a and radius b is vacuum. The outer sphere is non-conducting, and carries a surface charge density $\sigma_0 \cos\theta$. In the regions:



(I) $a < r < b$

(II) $b < r$

Find the potential Φ.

3.

A hollow grounded conducting sphere of radius R contains a point charge q at the point $a\hat{\mathbf{k}}$.

(a) Find the potential inside the sphere.

(b) Find the vector force on the charge q.

The sphere of the previous problem is now reduced to a conducting hemisphere, with a conducting flat base. The charge q is still at the point $a\hat{\mathbf{k}}$.

(e) Find the potential in the hemisphere.

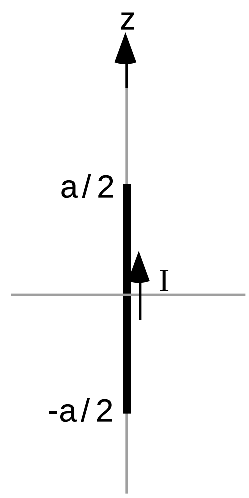
(f) Find the vector force on the charge q.

4. A plane wave of frequency ω, with $\vec{\mathbf{E}}_I = E_I \hat{j}$, is normally incident on a conducting plane of conductivity σ. The conducting plane fills the half space $z > 0$. The conductivity is very high, $\sigma >> \varepsilon\omega$, so the displacement current inside the conductor can be neglected. There is a reflected wave, $\vec{\mathbf{E}}_R$.

(a) Find the B and E fields inside the conductor in terms of their values at $z = 0$, as functions of z , σ and ω .

(b) Find the reflected electric field vector in terms of E_I , σ and ω .

5. An wire stretches along the z-axis from z = -a/2 to z = a/2. An alternating current of angular frequency ω runs in the wire, and the radiated EM wave has a wavelength λ that is much greater than the wire length a. A good approximation to the current density in the wire is



$$\vec{\mathbf{J}} = I_0 \delta(x)\delta(y) k\left(\frac{a}{2} - |z|\right) \sin(\omega t)\hat{\mathbf{k}} \quad ,$$

where k = ω/c .

At large distances r from the wire, where r >> a , and r >> λ ,

find as functions of r, θ, φ , ω and I_0 :

(a) The vector potential A .

(b) The magnetic field B.

(c) The electric field E,.

(d) Find the power radiated per unit solid angle as a function of θ, φ, ω and I_0 .

(e) Find the wire's electric dipole moment **p**, and its magnetic dipole moment **m** .